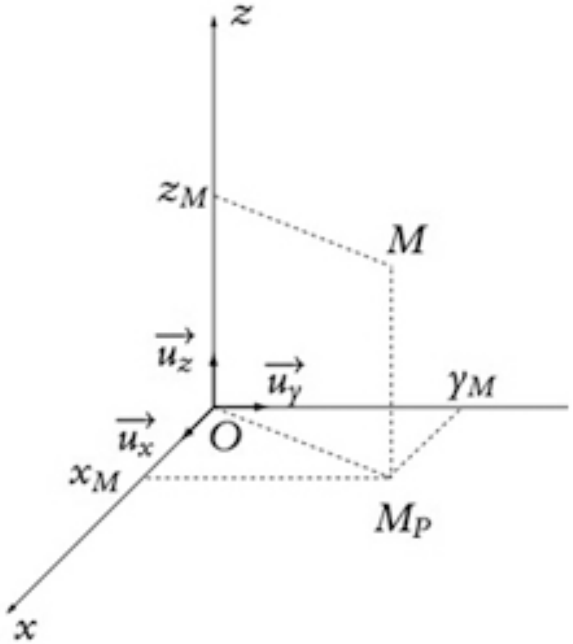

Figure 23.1 Coordonnées cartésiennes.

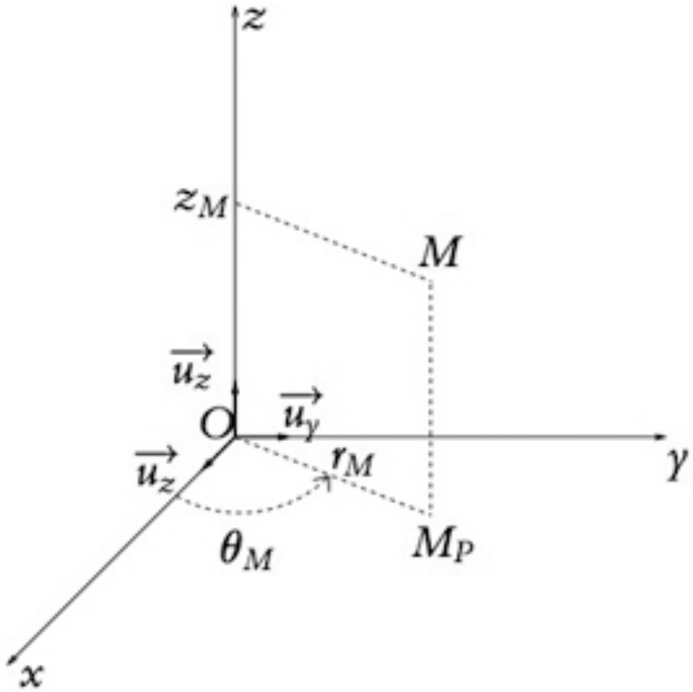

Figure 23.3 Coordonnées

cylindrique

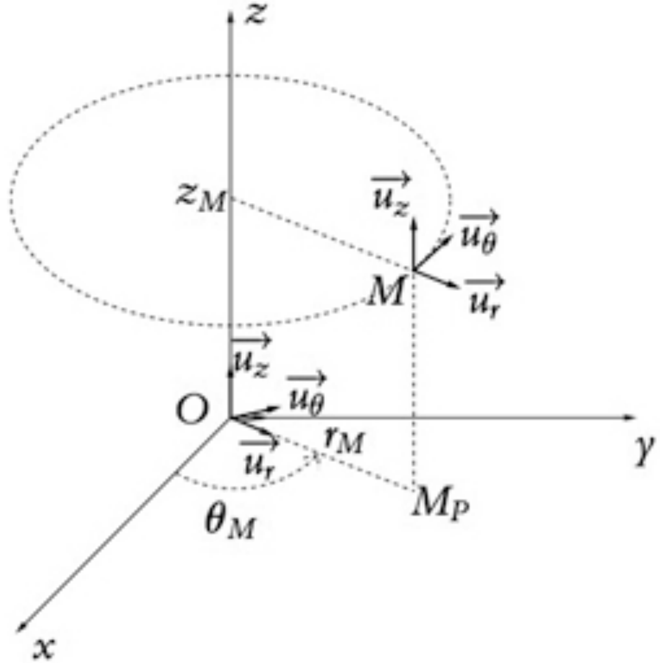

Figure 23.4 Base locale

sphérique

Construction d'une nouvelle base et représentation de la base CYLINDRIQUE

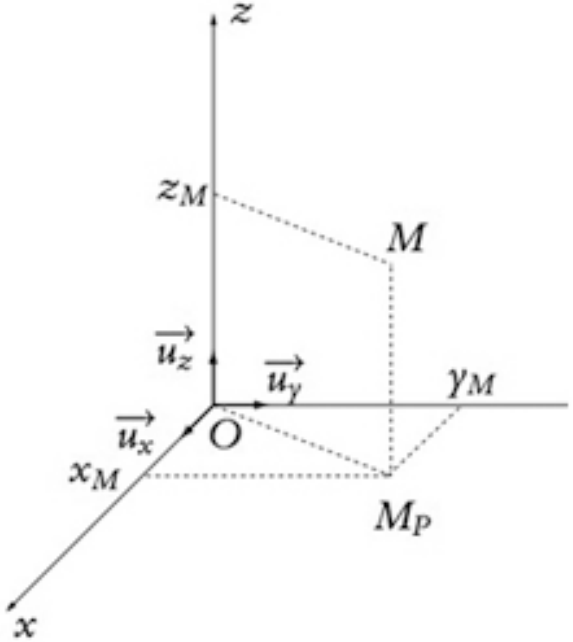

Figure 23.1 Coordonnées cartésiennes.

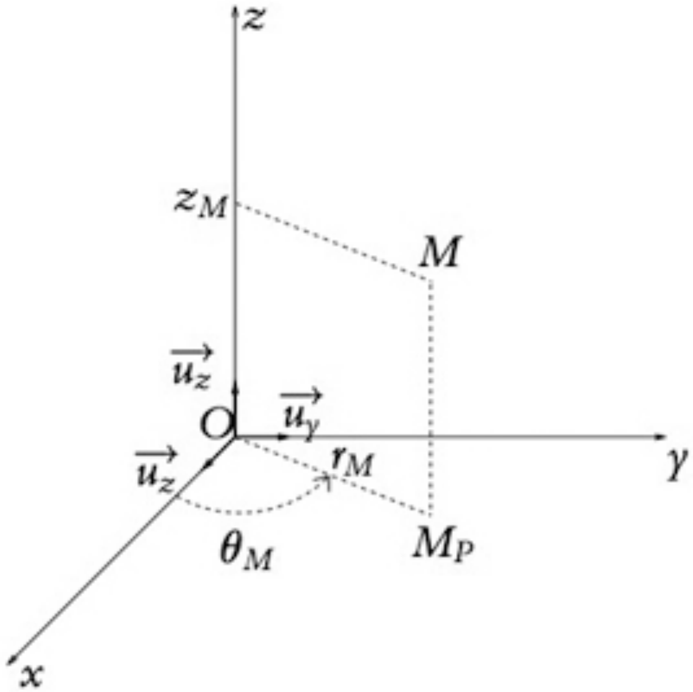

Figure 23.3 Coordonnées

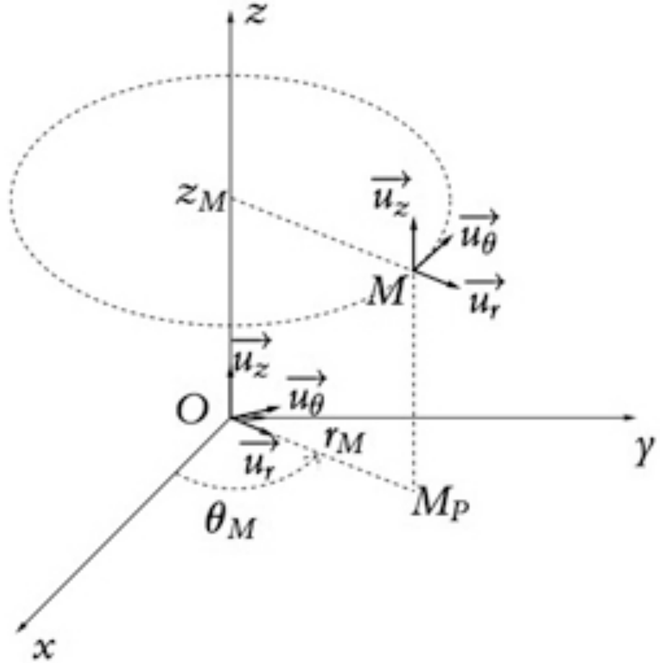

Figure 23.4 Base locale

$$\overrightarrow{OM} = r\overrightarrow{u_r} + z\overrightarrow{u_z}$$

$$\begin{aligned}
\vec{v} &= \frac{\mathrm{d}\overrightarrow{OM}}{\mathrm{d}t} \\
&= \dot{r}\overrightarrow{u_r} + r\frac{\mathrm{d}\overrightarrow{u_r}}{\mathrm{d}t} + \dot{z}\overrightarrow{u_z} \\
&= \dot{r}\overrightarrow{u_r} + r\frac{\mathrm{d}\overrightarrow{u_r}}{\mathrm{d}\theta}\frac{\mathrm{d}\theta}{\mathrm{d}t} + \dot{z}\overrightarrow{u_z} \\
&= \dot{r}\overrightarrow{u_r} + r\dot{\theta}\overrightarrow{u_\theta} + \dot{z}\overrightarrow{u_z}
\end{aligned}$$

$$\begin{aligned}
\vec{a} &= \frac{\mathrm{d}\vec{v}}{\mathrm{d}t} \\
&= \ddot{r}\overrightarrow{u_r} + \dot{r}\frac{\mathrm{d}\overrightarrow{u_r}}{\mathrm{d}t} + \dot{r}\dot{\theta}\overrightarrow{u_\theta} + r\frac{\mathrm{d}\dot{\theta}}{\mathrm{d}t}\overrightarrow{u_\theta} + r\dot{\theta}\frac{\mathrm{d}\overrightarrow{u_\theta}}{\mathrm{d}t} + \ddot{z}\overrightarrow{u_z} \\
&= \ddot{r}\overrightarrow{u_r} + \dot{r}\frac{\mathrm{d}\overrightarrow{u_r}}{\mathrm{d}\theta}\frac{\mathrm{d}\theta}{\mathrm{d}t} + \dot{r}\dot{\theta}\overrightarrow{u_\theta} + r\ddot{\theta}\overrightarrow{u_\theta} + r\dot{\theta}\frac{\mathrm{d}\overrightarrow{u_\theta}}{\mathrm{d}\theta}\frac{\mathrm{d}\theta}{\mathrm{d}t} + \ddot{z}\overrightarrow{u_z} \\
&= \ddot{r}\overrightarrow{u_r} + \dot{r}\overrightarrow{u_\theta}\dot{\theta} + \dot{r}\dot{\theta}\overrightarrow{u_\theta} + r\ddot{\theta}\overrightarrow{u_\theta} + r\dot{\theta}\left(-\overrightarrow{u_r}\right)\dot{\theta} + \ddot{z}\overrightarrow{u_z} \\
&= \left(\ddot{r} - r\dot{\theta}^2\right)\overrightarrow{u_r} + \left(2\dot{r}\dot{\theta} + r\ddot{\theta}\right)\overrightarrow{u_\theta} + \ddot{z}\overrightarrow{u_z}
\end{aligned}$$

Construction d'une nouvelle base et représentation de la base SPHERIQUE

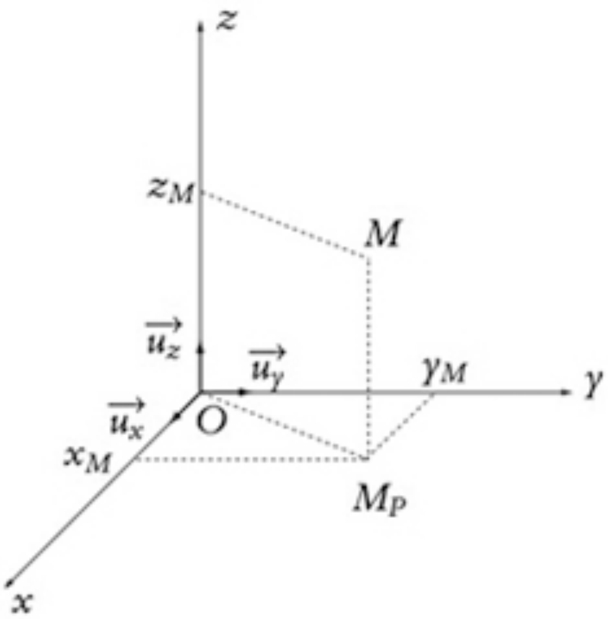

Figure 23.1 Coordonnées cartésiennes.

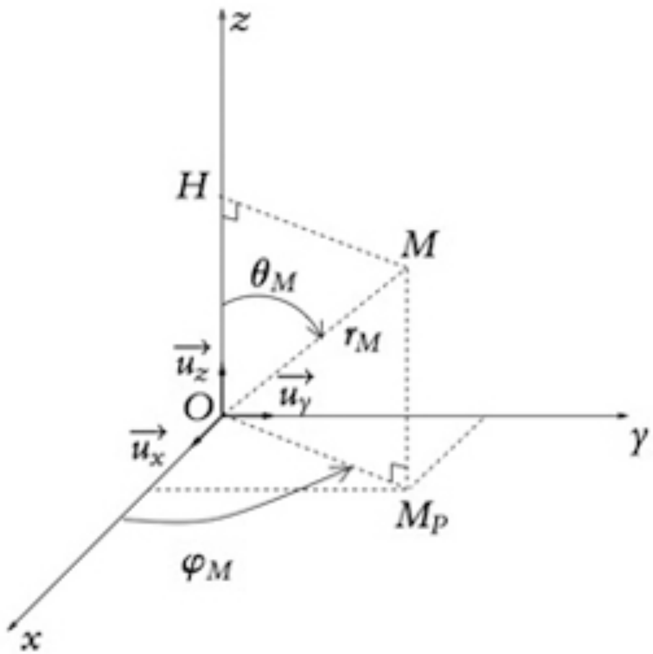

Figure 23.6 Coordonnées sphériques.

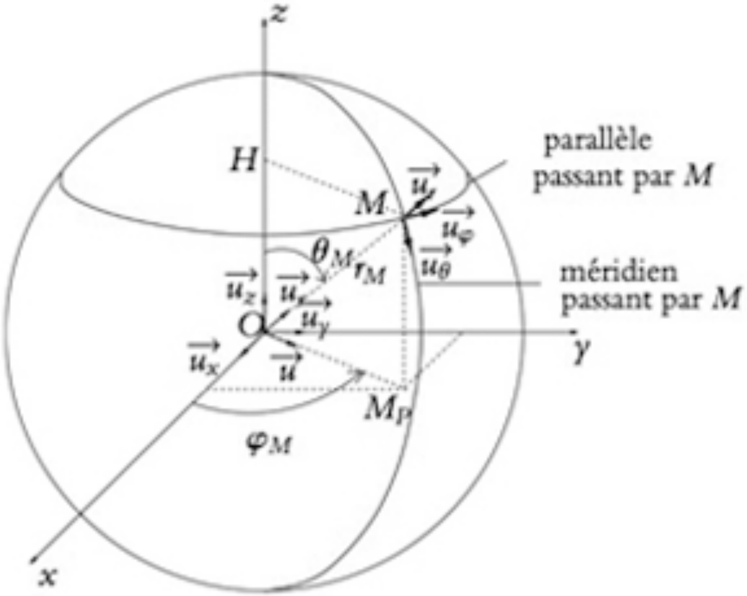

Figure 23.7 Base locale sphérique.

$$\overrightarrow{OM} = r(t)\,\vec{u_r}(t)$$

$$\begin{aligned}
\vec{v} &= \frac{\mathrm{d}\overrightarrow{OM}}{\mathrm{d}t} \\
&= \dot{r}\vec{u_r} + r\frac{\mathrm{d}\vec{u_r}}{\mathrm{d}t} \\
&= \dot{r}\vec{u_r} + r\frac{\partial\vec{u_r}}{\partial\theta}\frac{\partial\theta}{\partial t} + r\frac{\mathrm{d}\vec{u_r}}{\mathrm{d}\varphi}\frac{\mathrm{d}\varphi}{\mathrm{d}t} \\
&= \dot{r}\vec{u_r} + r\dot{\theta}\vec{u_\theta} + r\sin\theta\dot{\varphi}\vec{u_\varphi}
\end{aligned}$$

$$\begin{aligned}
\vec{a} &= \frac{\mathrm{d}\vec{v}}{\mathrm{d}t} \\
&= \left(\ddot{r} - r\dot{\theta}^2 - r\sin^2\theta\dot{\varphi}^2\right)\vec{u_r} + \left(2\dot{r}\dot{\theta} + r\ddot{\theta} - r\sin\theta\cos\theta\dot{\varphi}^2\right)\vec{u_\theta} \\
&\quad + \left(2\dot{r}\sin\theta\dot{\varphi} + r\sin\theta\ddot{\varphi} + 2r\cos\theta\dot{\theta}\dot{\varphi}\right)\vec{u_\varphi}
\end{aligned}$$